

# Hadronization in polarized semi-inclusive DIS: the question of independent fragmentation

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**Abstract.** A new formalism for the description of (un)polarized semi-inclusive deep inelastic scattering (DIS) at moderate energies is developed. Hadron production is modeled as a product of distribution functions and hadronization functions (HFs) weighted by the hard scattering cross sections, as has been done in the LEPTO event generator. The correct treatment of polarization effects shows that the description of semi-inclusive DIS (SIDIS) within this formalism includes a new non-perturbative input: *polarized* HFs. It is shown that this approach does not correspond to that commonly adopted with the independent quark fragmentation. The purity method used by the HERMES collaboration mixes the two approaches and ignores the contributions from polarized HFs. This method cannot be considered a precise tool for the extraction of polarized quark distributions from measured SIDIS asymmetries.

## 1 Introduction

Deep inelastic scattering (DIS) is one of the main sources of our knowledge of the nucleon structure. More information about both the nucleon structure and a hadron-production mechanism can be obtained by studying the semi-inclusive DIS (SIDIS).

It is evident that the theoretical description of SIDIS is much more complicated than that of DIS owing to our poor knowledge of the non-perturbative hadronization mechanism. Traditionally, one distinguishes two regions for hadron production: the current fragmentation region,  $x_F > 0$  and the target fragmentation region,  $x_F < 0$ <sup>1</sup>. The common assumption is that hadrons in the current fragmentation region with  $z > 0.2$  are produced in the independent quark fragmentation. Then, in the leading-order (LO) approximation of perturbative quantum chromodynamics (QCD) the SIDIS cross section for unpolarized target is given as

$$\sigma^h(x, z, Q^2) \propto (1 + (1-y)^2) \sum_q e_q^2 q(x, Q^2) D_q^h(z, Q^2) \quad (1)$$

and for the polarized beam and target

$$\Delta\sigma^h(x, z, Q^2) \propto (1 - (1-y)^2) \sum_q e_q^2 \Delta q(x, Q^2) D_q^h(z, Q^2). \quad (2)$$

The virtual photon asymmetry for production of the hadron  $h$  can be expressed as

$$A_1^h(x, z, Q^2) = \frac{\sum_q e_q^2 \Delta q(x, Q^2) D_q^h(z, Q^2)}{\sum_q e_q^2 q(x, Q^2) D_q^h(z, Q^2)}. \quad (3)$$

This equation can be rewritten as follows:

$$A_1^h(x, z, Q^2) = \sum_q \mathcal{P}_q^h(x, z, Q^2) \frac{\Delta q(x, Q^2)}{q(x, Q^2)}, \quad (4)$$

where the quark polarizations ( $\Delta q/q$ ) are factored out and the *purities*,  $\mathcal{P}_q^h$ , are defined as

$$\mathcal{P}_q^h(x, z, Q^2) = \frac{e_q^2 q(x, Q^2) D_q^h(z, Q^2)}{\sum_{q'} e_{q'}^2 q'(x, Q^2) D_{q'}^h(z, Q^2)}. \quad (5)$$

Recently, the important issue of the extraction of polarized quark distributions was again addressed by the HERMES collaboration [1]. They have used the above LO description of SIDIS and calculated purities using the Monte Carlo unpolarized event generator LEPTO [2]. Then, using the asymmetries measured for different hadrons, the helicity distributions  $\Delta q(x)$  were extracted by solving (4).

The main assumption of this method is that the hadronization mechanism in LEPTO is the same as in the naïve picture of SIDIS where all hadrons in the current fragmentation region with  $z > 0.2$  are produced in independent quark fragmentation and there are no additional terms in the numerator or the denominator of (3). This assumption is based on the factorization theorem of QCD

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<sup>1</sup> We use the standard SIDIS notations and variables, as in [1].

which is proven as an asymptotic statement for very-high-momentum transfers and hence very high energies.

An alternative approach is adopted in the LEPTO [2] event generator. The hadronization mechanism of this generator is based on the Lund string fragmentation model implemented in the JETSET program [3]. In this model the QCD confinement, the quantum numbers and energy-momentum conservation are taken into account. As a consequence, at moderate beam energies when the final hadronic system has a limited invariant mass,  $\sim 3\text{--}5\text{ GeV}$ , one cannot neglect the influence of the target remnant state on the distributions of hadrons produced in the current fragmentation region.

In Sect. 2 of this paper it will be demonstrated that the properties of the quark fragmentation functions extracted from generated LEPTO samples are in contradiction with generally accepted properties of independent quark fragmentation. The reason for this discrepancy is indicated. The generalization of the parton model expression for polarized SIDIS is given in Sect. 3. Finally, in Sect. 4 some discussion and conclusions are presented.

## 2 LEPTO and fragmentation functions

In the standard picture of SIDIS (see (1)) the quark fragmentation functions by definition depend on the type of hadron, quark flavor and fraction of quark energy carried by the hadron  $z$ , (there is also a weak dependence on  $Q^2$  due to perturbative QCD effects), and are *independent* of (a) the Bjorken variable  $x$  and (b) the target type. These properties related to the universality of fragmentation functions are essential; they indicate that one is dealing with independent quark fragmentation and that there is no influence of the target remnant on hadron production in the current fragmentation region.

These fragmentation functions are not well known for different hadron and quark types and the LEPTO event generator is used by the HERMES collaboration [1] to calculate the purities. However, hadronization in this generator is based on the Lund string fragmentation model and one has first to check if the issues (a) and (b) are satisfied in this approach. To this end samples of SIDIS events were generated for HERMES experimental conditions using the settings of LEPTO as in [1] (see also reference [64] of [1]). The option LST(8)=0 of LEPTO was used since it corresponds to the LO approximation of SIDIS<sup>2</sup>. The following cuts are used:  $Q^2 > 1\text{ GeV}^2$ ,  $W^2 > 10\text{ GeV}^2$ ,  $y < 0.85$ ,  $0.023 < x < 0.6$ ,  $E' > 3.5\text{ GeV}$ ,  $z > 0.2$ , and  $x_F > 0.1$ .

In Fig. 1 the quark fragmentation functions to  $\pi^+$ , extracted from the sample generated for the HERMES kin-

matics on a proton target, are presented as a function of  $z$ . The available range of the Bjorken variable,  $x$ , was divided into two equally populated intervals: (1)  $x < 0.094$  and (2)  $x > 0.094$ . As one can see from Fig. 1, the extracted quark fragmentation functions happen to be strongly dependent on the Bjorken  $x$  variable, in striking contradiction to the property (a) mentioned above. Note, that this cannot be attributed to the (weak)  $Q^2$ -dependence of the fragmentation functions. To demonstrate this the LO fragmentation functions from [4], which includes the QCD evolution, are also presented in the same figure for mean values of  $Q^2$  corresponding to Bjorken variable intervals: (1)  $Q^2 = 1.5(\text{GeV}/c)^2$  and (2)  $Q^2 = 3.4(\text{GeV}/c)^2$ .

The quark fragmentation functions obtained from the samples generated for proton and neutron targets with the cut  $x > 0.1$  are presented in Fig. 2. We see a dependence on the target type which contradicts property (b) of the fragmentation functions.

Such behavior of the fragmentation functions extracted from the generated samples is also observed for the production of other types of light meson such as  $\pi^-$ ,  $K^+$ ,  $K^-$  etc.

At this point one can conclude that the quark fragmentation functions extracted from the samples generated for HERMES kinematical conditions do not correspond to the commonly used notion of fragmentation function.

Let us recall that hadronization in the LEPTO event generator is based on string fragmentation and as it is stressed in [3]: “*the primary hadrons produced in string fragmentation come from the string as a whole, rather than from an individual parton*”. In other words the distributions of the produced hadrons retains the memory not only of the struck quark type but also of the target remnant and, hence, the entire string configuration.

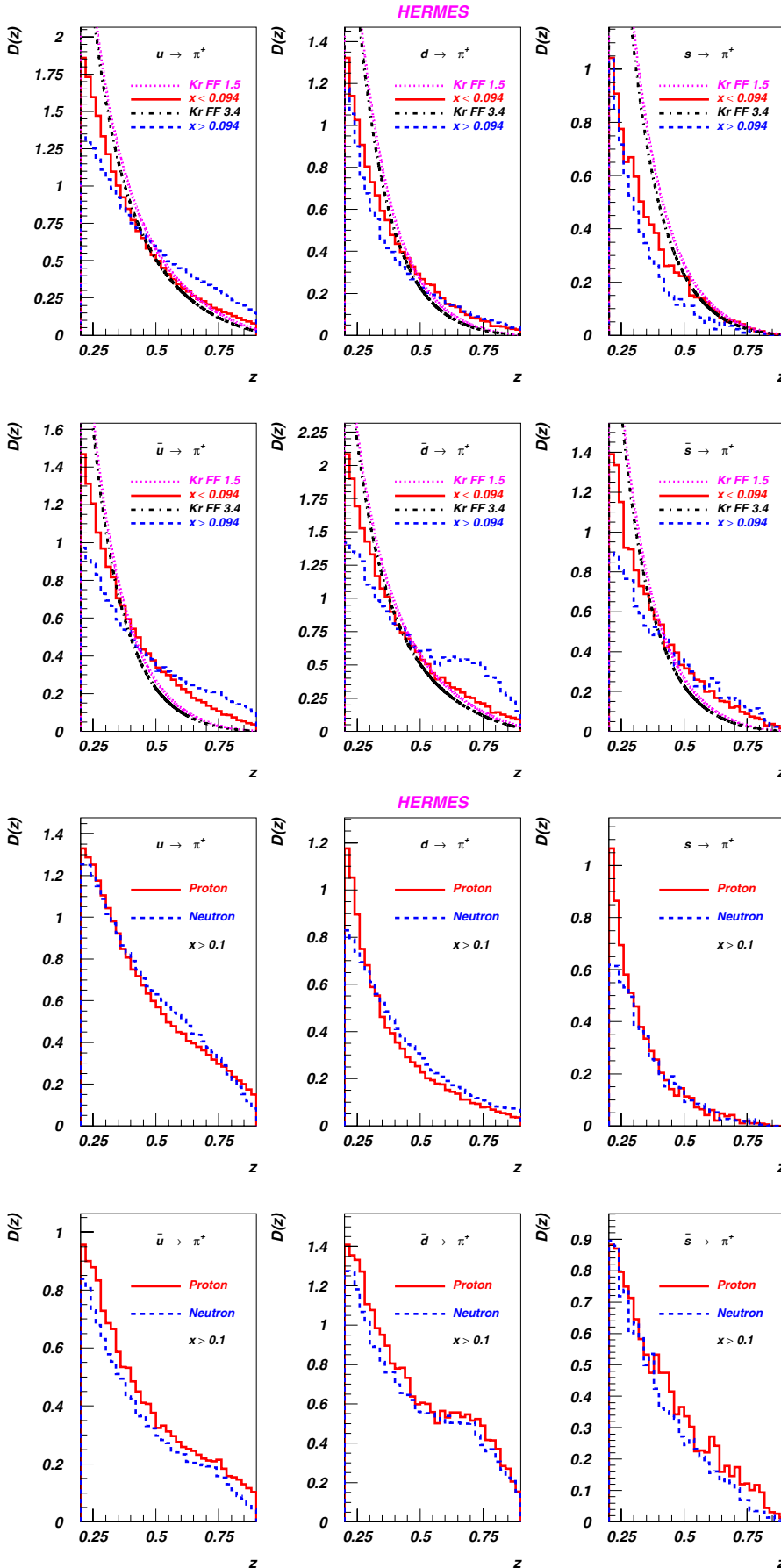
Event generation in LEPTO proceeds in three steps: first, the hard scattering kinematics ( $x, Q^2$ ) is chosen from the differential DIS cross section. Second, the struck quark flavor is chosen. Third, the string is set up and hadronized according to the Lund model implemented in the JETSET program [3]. Within this approach the SIDIS cross section can be expressed as

$$\sigma_N^h(x, z, Q^2) \propto (1 + (1 - y)^2) \sum_q e_q^2 q(x, Q^2) H_{q/N}^h(x, z, Q^2) \quad (6)$$

where the functions  $H_{q/N}^h(x, x_F, Q^2)$  describe the *conditional* probability of hadron  $h$  production in the hadronization of the system formed by the struck quark  $q$  and the corresponding target remnant. Let us call these the *hadronization functions* HFs. The target remnant type, and hence the whole fragmenting system configuration, depends both on the nucleon type and on the struck quark type<sup>3</sup>. This

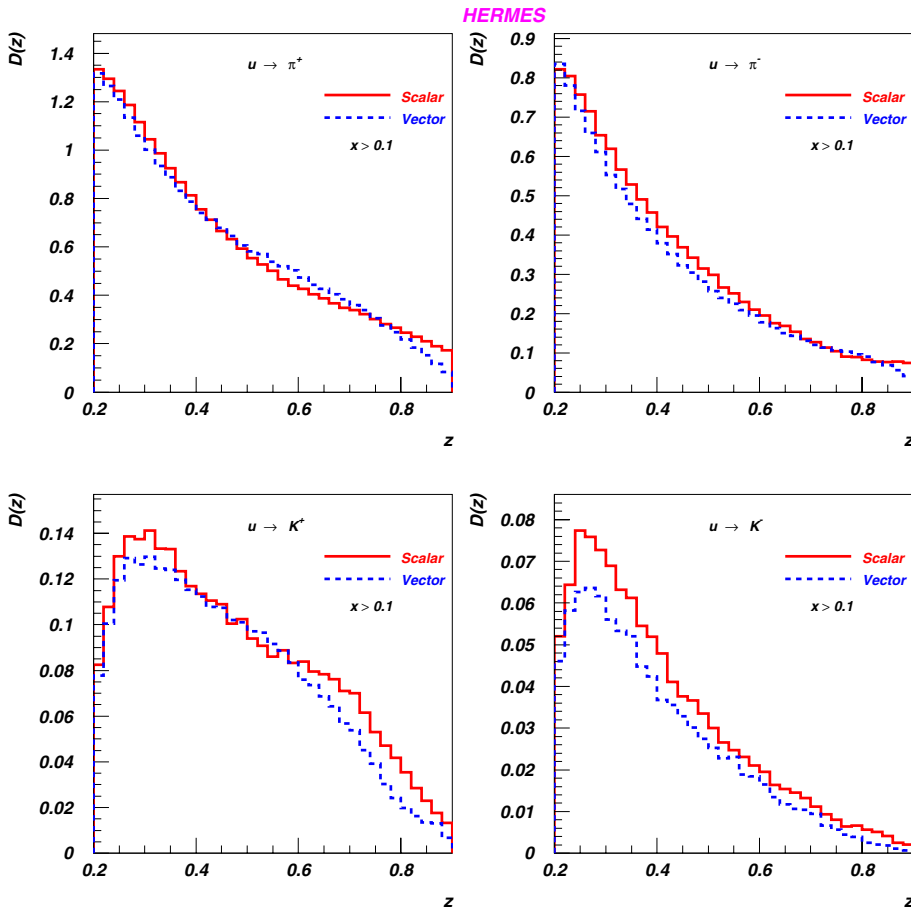
<sup>2</sup> The option LST(8)=1 used in reference [64] of [1] corresponds to inclusion in the generation of the first-order QCD matrix elements for gluon radiation and photon-gluon fusion. With this option (1) has to be replaced with the next-to-leading-order (NLO) expression to include the gluon distribution and fragmentation functions, which are not involved in the LO purity analysis. However, it was checked that the conclusions of the present paper remain valid independently of the choices of the LEPTO settings at the HERMES energy.

<sup>3</sup> The fragmenting system in LEPTO/JETSET is in general more complicated than a quark-diquark string. The target remnant state depends on the removed active parton type and the whole fragmenting system may contain multi-string configurations [2, 3].



**Fig. 1.** Quark fragmentation functions to  $\pi^+$  calculated for HERMES conditions. Solid line with cut  $x < 0.094$ ; dashed line with cut  $x > 0.094$ . Fragmentation functions from Kretzer: dotted line for  $Q^2 = 1.5 \text{ (GeV/c)}^2$ , dot-dashed line for  $Q^2 = 3.4 \text{ (GeV/c)}^2$

**Fig. 2.** Quark fragmentation functions to  $\pi^+$  calculated for HERMES conditions with cut  $x > 0.1$ . Solid line: proton target; dashed line: neutron target



**Fig. 3.** Dependence of the quark fragmentation functions on diquark type calculated for HERMES conditions with cut  $x > 0.1$ . Solid line: scalar diquark; dashed line: vector diquark

means that, in contrast with independent fragmentation functions, the HF's are non-universal – they depend on the process type and energy.

Note, that (6) is valid not only in the current fragmentation region but in the whole  $x_F$  interval.

The product  $q(x, Q^2) H_{q/N}^h(x, z, Q^2)$  is the probability to find the parton  $q$  in the nucleon,  $N$ , and, after hard interaction, to create a hadron  $h$  in the string hadronization. By its physical meaning (probabilistic interpretation) this object represents nothing but the fracture functions discussed in [5]. There exist certain arguments based on hand-bag diagram dominance that this concept may be applied even in the current fragmentation region of SIDIS [6]. The LEPTO/JETSET Monte Carlo program can be considered as a model for these functions. It is clear that from the generated samples one can actually extract only HF's and, as one can see from Fig. 1 and Fig. 2, even in the current fragmentation region one cannot neglect the dependence of this functions on the Bjorken  $x$  variable and on the target type.

As a further consideration of the target remnant state influence on the quark fragmentation functions let us now consider the spin inside LEPTO. In the simplest case when the valence  $u$ -quark is removed by hard scattering from a proton, the target remnant is a scalar,  $(ud)_0$ , diquark with probability  $w_0 = 0.75$  or a vector,  $(ud)_1$ , diquark with relative probability  $w_1 = 1 - w_0 = 0.25$  [2, 3]. In Fig. 3 the fragmentation functions for  $h = \pi^+, \pi^-, K^+$  and  $K^-$  extracted with  $x > 0.1$  cut are presented for the cases

when the target remnant diquark is chosen to be a 100% scalar ( $w_0 = 1$ ) or a vector ( $w_0 = 0$ ). We see that the HF's calculated with our generated samples already exhibit dependence (at the 5–10% level) on the target remnant spin state in unpolarized SIDIS.

This dependence has a quite general origin. To understand the underlying physics let us consider, as an example, a  $K^+$  meson production. At some stage of hadronization the  $\bar{s}s$ -pair is created and the  $s$ -quark can combine with the remnant diquark to form a strange baryon. The formation of a first-rank  $\Lambda$  hyperon is possible for a scalar diquark remnant and forbidden for the case of a vector diquark. In the second case only a heavier strange hyperon can be formed at first rank. Then, due to energy-momentum conservation, the available energy for a  $K^+$  meson production will be less in the second case compared with the first case. This is the reason why in Fig. 3 the  $u$ -quark fragmentation functions into  $K^+$  is higher for scalar diquark sample.

### 3 Polarized SIDIS and string fragmentation

In the ordinary factorized picture the same *unpolarized* fragmentation function is entering into the expression for the cross section of the unpolarized and polarized SIDIS. Let us now compare what will happen if we include spin and generalize the picture of the HF's for SIDIS.

At present there is no the polarized version of the string-fragmentation Monte Carlo program for event generation. It is evident that the description of polarized SIDIS is more complicated than in the unpolarized case. Let us, as an example, again consider the simplest case when the valence  $u$ -quark with positive ( $u^+$ ) or negative ( $u^-$ ) helicity is removed from the nucleon with positive or negative helicity,  $N^+$  or  $N^-$ . Within the SU(6) quark-diquark model used in [2, 3] the polarized nucleon wavefunctions are given as

$$p^+ = \frac{1}{\sqrt{18}} \left\{ u^+ [3(ud)_{0,0} + (ud)_{1,0}] - \sqrt{2} u^- (ud)_{1,1} - \sqrt{2} d^+ (uu)_{1,0} + 2d^- (uu)_{1,1} \right\}, \quad (7)$$

$$n^+ = \frac{1}{\sqrt{18}} \left\{ d^+ [3(ud)_{0,0} + (ud)_{1,0}] - \sqrt{2} d^- (ud)_{1,1} - \sqrt{2} u^+ (dd)_{1,0} + 2u^- (dd)_{1,1} \right\}, \quad (8)$$

where  $(q_1 q_2)_{(i,k)}$  stands for the diquark formed by the  $q_1$ - and  $q_2$ -quarks with spin  $i$  and spin projection  $k$ .

Using the explicit form of the polarized nucleon wavefunctions one can calculate the relative probabilities,  $w$ , of the different target remnant states (and hence the states of the entire string) depending on the struck quark and the nucleon polarizations. For example, when the  $u^+$ -quark is removed from the  $p^+$  we get the following string configurations with corresponding probabilities  $w$

$$p^+ \ominus u^+ \implies \begin{cases} \{(ud)_{0,0} \dots u^+\}, & w = 0.9, \\ \{(ud)_{1,0} \dots u^+\}, & w = 0.1, \end{cases} \quad (9)$$

where  $\{(q_1 q_2)_{i,k} \dots q^+\}$  denotes the string formed by the struck quark  $q^+$  and the diquark  $(q_1 q_2)_{i,j}$ . Similarly, when the  $u^+$ -quark is removed from  $p^-$  we get

$$p^- \ominus u^+ \implies \{(ud)_{1,-1} \dots u^+\}, \quad w = 1. \quad (10)$$

For the neutron target we have

$$n^+ \ominus u^+ \implies \{(dd)_{1,0} \dots u^+\}, \quad w = 1, \quad (11)$$

and

$$n^- \ominus u^+ \implies \{(dd)_{1,-1} \dots u^+\}, \quad w = 1. \quad (12)$$

The relations (9–12) demonstrate that the string configuration indeed depends not only on the struck quark type and its polarization but also on the target type and polarization. As we have seen in Sect. 2 the description of hadron production in the current fragmentation region of SIDIS within the Lund fragmentation model does not correspond exactly to the commonly adopted simple picture of independent quark fragmentation but rather to the more complicated approach based on fracture functions. Even in unpolarized SIDIS, HF's depend on the target fragment spin states, as is demonstrated in Fig. 3. Then in polarized SIDIS the dependence on the target and struck quark polarizations appears. So, one has to generalize (6) for the polarized SIDIS case.

Let us start with the SIDIS cross section  $\sigma_{N\lambda_l\lambda_N}^h$  for the positive helicity lepton,  $\lambda_l = +1$  and hadron,  $\lambda_N = +1$ :

$$\sigma_{N^{++}}^h \propto \sum_q e_q^2 \left\{ q^+ H_{q/N^{++}}^h + (1-y)^2 q^- H_{q/N^{-+}}^h \right\}, \quad (13)$$

where  $H_{q/N\lambda_q\lambda_N}^h$  describes the production probability of the hadron  $h$  in the quark-target remnant system fragmentation and depends not only on  $x$  and  $z$  (or  $x$  and  $x_F$ ) but also on the struck quark and nucleon helicities,  $\lambda_q = \pm 1$  and  $\lambda_N = \pm 1$ . Similarly

$$\sigma_{N^{+-}}^h \propto \sum_q e_q^2 \left\{ q^- H_{q/N^{+-}}^h + (1-y)^2 q^+ H_{q/N^{--}}^h \right\}. \quad (14)$$

The partially polarized beam state,  $l^{\lambda_l}$ , (with helicity  $\lambda_l$ ) can be described as  $l^{\lambda_l} = 1/2(1 + \lambda_l)l^+ + 1/2(1 - \lambda_l)l^-$  and similarly for the nucleon. Then, in the general case of arbitrary polarized beam and target, we have

$$\begin{aligned} \sigma_{N\lambda_l\lambda_N}^h &\propto (1 + \lambda_l)(1 + \lambda_N)\sigma_{N^{++}}^h \\ &\quad + (1 + \lambda_l)(1 - \lambda_N)\sigma_{N^{+-}}^h \\ &\quad + (1 - \lambda_l)(1 + \lambda_N)\sigma_{N^{-+}}^h \\ &\quad + (1 - \lambda_l)(1 - \lambda_N)\sigma_{N^{--}}^h. \end{aligned} \quad (15)$$

Now, using the relations  $H_{q/N^{++}}^h = H_{q/N^{--}}^h$  and  $H_{q/N^{+-}}^h = H_{q/N^{-+}}^h$  which follow from parity invariance and introducing

$$H_{q/N}^h = H_{q/N^{++}}^h + H_{q/N^{+-}}^h, \quad (16)$$

$$\Delta H_{q/N}^h = H_{q/N^{++}}^h - H_{q/N^{+-}}^h$$

after simple algebra one gets:

$$\begin{aligned} \sigma_{N\lambda_l\lambda_N}^h &\propto [1 + (1-y)^2] \sum_q e_q^2 \left\{ q H_{q/N}^h + \Delta q \Delta H_{q/N}^h \right\} \\ &\quad + \lambda_l \lambda_N [1 - (1-y)^2] \\ &\quad \times \sum_q e_q^2 \left\{ \Delta q H_{q/N}^h + q \Delta H_{q/N}^h \right\}, \end{aligned} \quad (17)$$

where now  $\lambda_l$  and  $\lambda_N$  are the (arbitrary) beam and target helicities.

Equation (17) is very similar to the equation proposed in [7]. The difference is that functions  $H_{q/N}^h$  and  $\Delta H_{q/N}^h$  are not independent quark fragmentation functions like in [7]. It is well known that the single spin dependence is forbidden by parity invariance for independent fragmentation integrated over transverse momentum. For this reason the same quark fragmentation functions enter in (1–2). In contrast, HF's describe the probability of hadron production in the hadronization of the whole *struck quark-target remnant* system and, hence, one deals with the double spin effects. Even for hadrons produced in the current fragmentation region these functions can depend not only on the fraction of the quark energy carried by produced hadron but also

on the whole hadronic CMS energy and *the target and the struck quark polarizations*.

When integrated over the whole available phase space of the selected hadron, (17) transforms to the standard parton model expression for the polarized DIS, provided that the following sum rules hold:

$$\begin{aligned} \sum_h \int dz z H_{q/N}^h(x, z, Q^2) &= 1, \\ \sum_h \int dz z \Delta H_{q/N}^h(x, z, Q^2) &= 0. \end{aligned} \quad (18)$$

## 4 Discussion and conclusions

The standard expression for the SIDIS description in the current fragmentation region<sup>4</sup> is obtained if one assume that

$$H_{q/N}^h(x, z, Q^2) \rightarrow D_q^h(z, Q^2) \quad (19)$$

and

$$\Delta H_{q/N}^h(x, z, Q^2) \rightarrow 0. \quad (20)$$

As we have demonstrated in Sect. 2 relation (19) is not correct for the HERMES experimental conditions in the Lund fragmentation model. On the other hand we have seen in Fig. 3 that hadronization in this model depends on the target remnant spin quantum numbers. In the case of polarized SIDIS the relative probabilities of different target remnant states depend on the target and struck quark polarizations, see (9-10). So, there is no any reason to believe that the relation (20) will hold for the polarized SIDIS at moderate energies. Thus, a *new non-perturbative inputs – the polarized HFs*,  $\Delta H_{q/N}^h(x, z, Q^2)$ , are needed. In this case (3) underlying the purity method is not exact and must to be replaced by

$$\begin{aligned} A_1^h(x, z, Q^2) &= \\ \frac{\sum_q e_q^2 q(x, Q^2) H_{q/N}^h(x, z, Q^2) \left[ \frac{\Delta q(x, Q^2)}{q(x, Q^2)} + \frac{\Delta H_{q/N}^h(x, z, Q^2)}{H_{q/N}^h(x, z, Q^2)} \right]}{\sum_q e_q^2 q(x, Q^2) H_{q/N}^h(x, z, Q^2) \left[ 1 + \frac{\Delta q(x, Q^2) \Delta H_{q/N}^h(x, z, Q^2)}{q(x, Q^2) H_{q/N}^h(x, z, Q^2)} \right]}, \end{aligned} \quad (21)$$

with extra contributions in the numerator and denominator compared to (3). As one can see from Fig. 3 the second term in the square brackets in the numerator can reach 5–10%. Its influence can still be negligible in the unpolarized cross section (second term in square brackets of denominator), since it entering multiplied by the quark polarization; whereas, in the numerator, it can be comparable with (or even greater than) the first term.

Though we are not able to calculate from first principles the HFs, one can try to estimate the possible effects

<sup>4</sup> Note that these expressions are based on QCD factorization theorems, which represent asymptotic statements valid for very high lepton energies,  $Q^2$  and  $W$ . Only under these conditions can one neglect the influence of the target remnant on hadron production in the current fragmentation region.

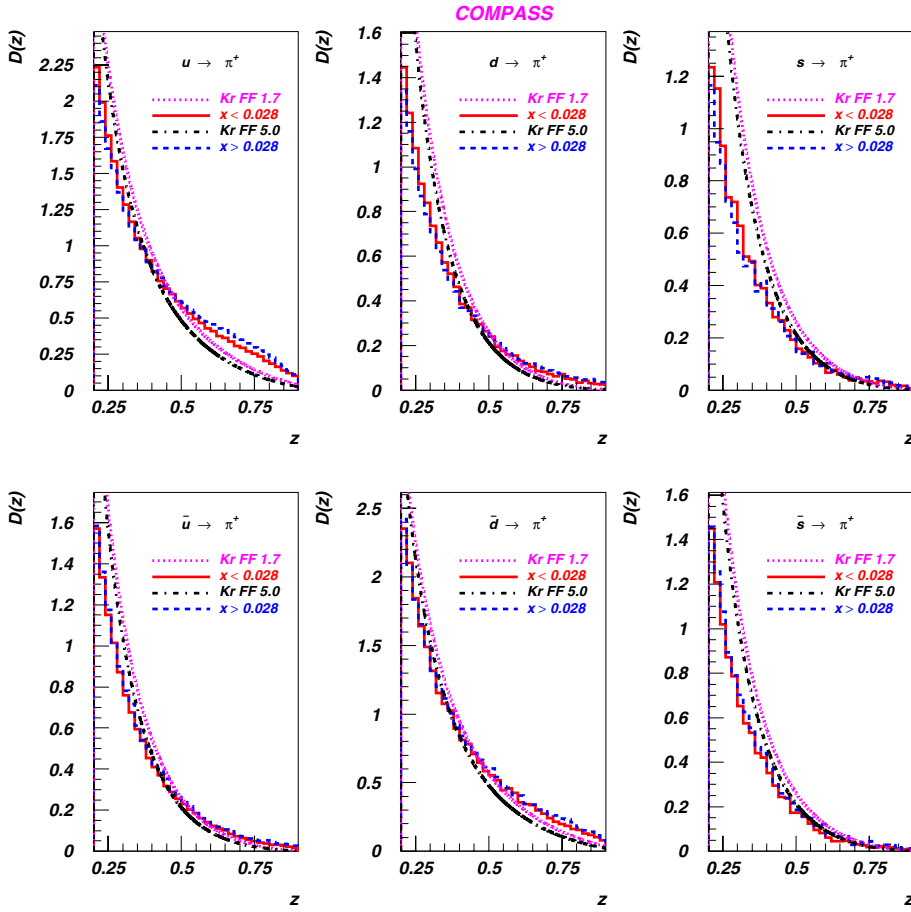
of polarized HFs on the extraction of polarized quark distribution. For example, one could estimate the effects of scalar and vector diquarks using the formalism of Sect. 2 and calculate  $\frac{\Delta H_{q/N}^h(x, z, Q^2)}{H_{q/N}^h(x, z, Q^2)}$  for each  $x$ -bin.

In [9] a model for the extra contribution in the numerator of (21) has been considered and it was demonstrated that our ignorance of polarized HFs may lead to incorrect results for polarized sea quark distributions. Here it is demonstrated that using LEPTO in analysis based on the factorized approach is inconsistent. As a consequence it is essential in using the factorization approach to polarized quark distribution extraction first to check it, since the polarization dependence is more sensitive to factorization than the unpolarized multiplicity distributions.

It is interesting to study how these effects depend on the energy and in particular how they will affect the COMPASS [8] analysis. In Fig. 4 the same distributions as in Fig. 1 are presented for COMPASS kinematics. One can see that the dependence on Bjorken variable of the quark fragmentation functions extracted from generated samples is less pronounced than in case of HERMES experimental conditions. The same observation is also valid for the dependence upon the target type and the target remnant spin state dependencies. This means that that polarized HFs may be negligible in the current fragmentation region at high energies.

The string configurations considered in (9–12) correspond to the simplest case of removing the valence quark from nucleon. In the case, when the virtual photon interacts with the sea quark or higher-order hard scattering processes are considered, the target remnant and the final parton configuration are more complicated [2]. For example, in photon-gluon fusion the target remnant is split into a quark and a diquark that form two respective separate strings with the antiquark and quark produced in the fusion process. One can generalize (17) to include the corrections from higher-order QCD hard processes. The generalized expression for the polarized cross section of single and two hadron productions will, in addition, contain new unknown HFs,  $\Delta H_{g/N}^h(x, z, p_T^h, Q^2)$  and  $\Delta H_{g/N}^{h_1, h_2}(x, z_1, z_2, p_T^{h_1}, p_T^{h_2}, Q^2)$ , with the corresponding distribution functions,  $(\Delta)g(x, Q^2)$ . This means that the validity of the Monte-Carlo-based approach [10] to extract the polarized gluon distribution may also be questionable at moderate energies.

It has been recently noted [6] that the appearance of separate distribution and fragmentation functions cannot be proven in general, but is rather assumed and justified a posteriori, while the natural framework to describe SIDIS involves fracture functions. These functions can be also generalized to describe the T-odd single-spin asymmetries [6]. As was mentioned in Sect. 2 within the Lund fragmentation framework the fracture functions can be represented by products of distribution functions and HFs. Recent developments in the theory of SIDIS for single-spin asymmetries and diffractive phenomena also show that one cannot neglect the interaction of (colored) removed partons and target remnants (see, for example, [11] and references therein,



**Fig. 4.** Quark fragmentation functions to  $\pi^+$  calculated for COMPASS conditions. Solid line: with the  $x < 0.028$ ; dashed line: with cut  $x > 0.028$ . Fragmentation functions from Kretzer: dotted line for  $Q^2 = 1.7(\text{GeV}/c)^2$ , dot-dashed line for  $Q^2 = 5.0(\text{GeV}/c)^2$

and [6]). This again indicates that the description of SIDIS based on the naïve parton model with the independent fragmentation is only an approximation, to be justified at moderate energies.

Let us stress that the approach developed in Sect. 3 is not directly derived from QCD but is similar to that used in LEPTO event generator. It assumes that in the DIS regime the hard scattering and the hadronization are factorized. As was mentioned a long time ago [12] the concept of independent fragmentation can be justified only when there is enough phase space for the final hadronic system. The important issue is not only high  $Q^2$  but also a large rapidity interval available for a given hadron production. In contrast to the independent fragmentation model, the Lund model deals with the whole final quark-target remnant hadronization and takes into account energy-momentum conservation, color flow and quantum number correlations. As a consequence, at moderate  $W$  there is a non-negligible influence of the target remnant state on the distributions of hadrons produced in the current fragmentation region, as explained at the end of Sect. 2.

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